

## CLAIMS

1. A process for haptic rendering of three-dimensional soft bodied objects for virtual interactions comprising the steps of:

forming a three dimensional occupancy map of voxels, forming the surface and

5 bound the object,

forming a multi-dimensional coordinate system for each point, the coordinate system defining a vertex center,

defining minimum and maximums distances of one voxel to all its neighboring voxels (occupied neighbor points),

10 defining multi-dimensional maximum offsets that an occupied point can maintain relative to its center,

detecting when the minimums or maximums of distance and/or offsets are violated, and in response thereto

15 moving the points in violation to locations relative to the neighboring occupied points and the points' centers that satisfy the minimum and maximum for distance and offsets,

repeating the detecting and moving steps for the entire occupancy map until there are essentially no violations.

20 2. The process as defined in claim 1 further comprising determining a minimum energy state for all occupied points, and continuing the repeating of the detection and moving steps until the entire object is at a minimum energy state.

25 3. The process as defined in claim 1, further comprising the steps of determining, from the relative moving of the points and the offsets from the center for each point, the direction and size of the force of the colliding bodies, and delivering that force in size and direction via a haptic device.

4. The process as defined in claim 1, wherein when one body collides with and penetrates or bounces from another, further comprising the steps of:

forming a proxy of the penetrating or bouncing body that maintains its position on the surface, and wherein the moving of the points responds to the proxy as well as the penetrating body.

5. The process as defined in claim 1 further comprising damping the responses of the points as collisions occur.

6. An apparatus for haptic rendering of three-dimensional soft bodied objects for virtual interactions comprising in combination:

means for forming a three dimensional occupancy map of voxels, forming the surface and bound the object,

means for forming a multi-dimensional coordinate system for each point, the coordinate system defining a vertex center,

means for defining minimum and maximums distances of one voxel to all its neighboring voxels (occupied neighbor points),

means for defining multi-dimensional maximum offsets that an occupied point can maintain relative to its center,

means for detecting when the minimums or maximums of distance and/or offsets are violated, and in response thereto

means for moving the points in violation to locations relative to the neighboring occupied points and the points' centers that satisfy the minimum and maximum for distance and offsets, and

repeating the detecting and moving steps for the entire occupancy map until there are no violations.

7. The apparatus as defined in claim 1 further comprising means for determining a minimum energy state for all occupied points, and for continuing the

repeating of the detection and moving steps until the entire object is at a minimum energy state.

8. The apparatus as defined in claim 1, further comprising the means for determining, from the relative moving of the points and the offsets from the center for  
5 each point, the direction and size of the force of the colliding bodies, and for delivering that force in size and direction via a haptic device.

9. The apparatus as defined in claim 1, constructed and arranged so that when one body collides with and penetrates or bounces from another, and a proxy of the penetrating or bouncing body is formed that maintains its position on the surface,  
10 and so that wherein the moving of the points responds to the proxy as well as the penetrating body.

10. The apparatus as defined in claim 1 further comprising means for damping the responses of the points as collisions occur.

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